AN INSIGHT INTO UWB INTERFERENCE FROM A SHOT NOISE PERSPECTIVE

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1. ABSTRACT

This paper considers the effects of UWB interference on narrowband systems from the perspective of that of a shot noise random process. Shot noise is perhaps best known as the time-dependent fluctuations in electrical current caused by the discrete nature of the electron charge in such devices as tunnel diodes, Schottky barrier diodes and P-N junctions. Here, the discrete nature of the UWB-excited impulse response of a victim receiver is shown, under mild conditions, to behave as shot noise. The statistical properties of the receiver output are then compared with measurement results obtained to date by the National Telecommunications and Information Administration (NTIA), Stanford University and the Department of Transportation (DOT) and others.

2. INTRODUCTION

In a recent set of comprehensive test reports [1-4], the U.S. NTIA demonstrated the potential for certain classes of UWB emissions to degrade the performance of Global Positioning Satellite (GPS) receivers and selected Federal Government radar systems operating below 3.1 GHz, even if the UWB systems radiated within existing FCC Part 15 emission limits. The reports suggested that high pulse repetition frequency (PRF) UWB emissions, with or without incidental pulse train dithering, had the highest potential for such interference. Similar results were shown in DOT-sponsored testing by Stanford University [5] and by the Johns Hopkins University analyses [6] of the University of Texas – Austin and Time Domain Corporation measurement study [7].

Most interesting was the demonstration that, for UWB PRFs exceeding the operational or resolution bandwidth (RBW) of the victim receiver, the power P_d detected by the victim receiver (relative to the total UWB power P_T at the receiver terminals) was proportional to the square of the ratio of the PRF to the total radiated bandwidth

$$P_d \approx \left(\frac{PRF}{B}\right)^2 P_T \quad \text{for PRF > RBW ;}$$

and that, for UWB PRFs *less than* the resolution bandwidth, the detected power was proportional to the ratio of the RBW to the total radiated bandwidth

$$P_d \approx \left(\frac{RBW}{B}\right)^2 P_T \quad \text{for PRF < RBW .}$$

In the former case, successive UWB pulses are effectively integrated within the victim receiver's filter; while in the latter, the UWB pulses remain separable and distinct within the receiver. These results, of course, have extremely important ramifications for the future commercialization of high data rate UWB communications applications for wireless local area (WLAN) and personal area networks (PANs)

Unfortunately, this phenomenon is not well understood. Indeed, the advantage that UWB has had in low probability of detection (LPD) systems has been due to the fact that such systems operate with extremely low pulse duty cycles δ -- a combination of extremely short pulse duration τ and low PRF ($\delta = \tau$ PRF). With extremely low pulse duty cycles, the average field strength intensity from such emissions is also very low. However, in commercial applications of high data rate (100's of Mb/s) UWB communications, pulse duty cycles can be quite large and, given the high PRF, the problem is further exacerbated by the above phenomenon.

In the following section, we provide a simple model for UWB interference, noting that the effect of a short pulse UWB emission on a victim receiver can be modeled as a shot noise process.

3. A SIMPLE STATISTICAL MODEL FOR UWB INTERFERENCE

Let us first assume that, at the input to the victim receiver, the UWB signal x(t) consists of a sequence of pulses positioned randomly in time; i.e.,

$$x(t) = \sum_{i=-\infty}^{\infty} p(t - t_i)$$

where p(t) is the received pulse impinging upon the antenna and $\{t_i\}$ are the pulse arrival times. By considering the $\{t_i\}$ as randomly distributed in time, the analysis is most appropriate for pseudo-randomly dithered UWB pulse trains.

Similar analyses can be performed for constant PRF waveforms; i.e., those for which $t_i = iT$ for some constant pulse repetition interval (PRI) T. In this case, a more appropriate model for the random process x(t) might be

$$x_1(t) = \sum_{i=-\infty}^{\infty} a_i\, p(t - iT)$$

where the $\{a_i\}$ are either {1,0} random variates (ON-OFF keying) or have some amplitude probability density function (PAM). Combinations of amplitude and time (dither) modulations can also be considered.

Further assume that the average pulse rate (pulse repetition frequency or PRF) for x(t) is given by the constant parameter λ. The more general case of $\lambda = \lambda(t)$ is also straightforward to analyze, and yields nearly identical results. A time-varying rate parameter can be used, for example, to more accurately model bursty UWB emissions (e.g., packetized transmissions).

Now, under mild assumptions [8], the random "event points" $\{t_i\}$ can be modeled as a Poisson random process with mean count rate λ. One can also write x(t) as

$$x(t) = \sum_{i=-\infty}^{\infty} u_{-1}(t - t_i) * p(t)$$

where $u_{-1}(t)$ is the unit impulse function, and where $\sum_{i=-\infty}^{\infty} u_{-1}(t - t_i)$ is a Poisson stream of impulses.

If $h_R(t)$ is the impulse response of the victim receiver, the receiver output y(t) to the UWB input excitation is given by the expression

$$y(t) = x(t) * h_R(t) = \sum_{i=-\infty}^{\infty} u_{-1}(t - t_i) * (p(t) * h_R(t)) \equiv \sum_{i=-\infty}^{\infty} h(t - t_i) \qquad (1)$$

where $h(t) \equiv p(t) * h_R(t)$ is the convolution of a single UWB pulse with the receiver impulse response. Note that for most problems of interest, $h(t) \approx h_R(t)$ as the bandwidth of the UWB pulse is significantly greater than that of $h_R(t)$. The physical model is shown in Figure 1 below.

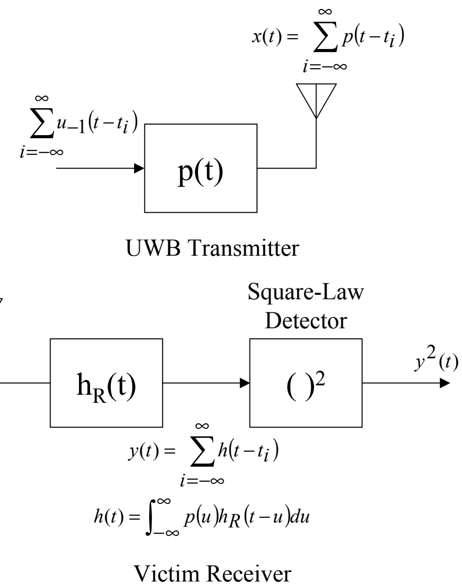

Figure 1. Shot Noise Model for UWB Excitation of Victim Receiver

As modeled in (1), the random process y(t) is a shot noise process whose statistical properties are well known [9]. For example,

$$E[y(t)] = \lambda \int_{-\infty}^{\infty} h(u)\,du$$

$$Var[y(t)] = \lambda \int_{-\infty}^{\infty} h^2(u)\,du$$

where E(.) denotes statistical expectation and Var(.) denotes statistical variance. For a bandpass system h(t), the expected value of the shot noise output is identically zero since

$$E[y(t)] = \lambda \int_{-\infty}^{\infty} h(u)\,du = \lambda H(0) = 0$$

where H(f) is the Fourier transform of h(t).

From an interference perspective, the statistics of the square-law detected output, i.e. $y^2(t)$, are of particular interest. For example, one can readily show [9, Chapter 16] that the autocorrelation function for the square-law detector output is given by the relationship:

$$E\{y^2(t+\tau)y^2(t)\} = \lambda \int_{-\infty}^{\infty} h^2(\tau+\alpha)h^2(\alpha)\,d\alpha + 2\lambda^2\left[\int_{-\infty}^{\infty} h(\tau+\alpha)h(\alpha)\,d\alpha\right]^2 + \lambda^2\left[\int_{-\infty}^{\infty} h^2(\alpha)\,d\alpha\right]^2 .$$

Evaluating the autocorrelation function at the origin, the energy in the square-law detected output is seen to be

$$Energy_{\text{detector output}} = E\{y^2(t)y^2(t)\}$$
$$= \lambda\int_{-\infty}^{\infty} h^4(\alpha)d\alpha + 3\lambda^2\left[\int_{-\infty}^{\infty} h^2(\alpha)d\alpha\right]^2 .$$

Now, if h(t) is the impulse response of the ideal bandpass filter

$$H(f) = \begin{cases} 1 & f_0 - B/2 \le f \le f_0 + B/2 \\ 1 & -f_0 - B/2 \le f \le -f_0 + B/2 \\ & 0 \text{ elsewhere} \end{cases};$$

then it is straightforward to show that

$$Energy_{\text{detector output}} = 4B^3\lambda + 12B^2\lambda^2$$
$$= 4B^4\left[\frac{\lambda}{B} + 3\left(\frac{\lambda}{B}\right)^2\right] .$$

Note that, as λ >> B, the energy in the square-law detector output increases quadratically with λ. For λ << B, this dependency is approximately linear in λ.

This relationship between detected energy in a victim receiver and the incident UWB pulse repetition frequency was observed in UWB interference testing by NTIA [1-4], Stanford University and the Department of Transportation [5], and the University of Texas – Austin and Johns Hopkins University Applied Physics Laboratory analyses of Time Domain Corporation UWB equipment [6-7].

4. UWB GAUSSIAN APPROXIMATION

It is of particular interest to consider the case when the UWB pulse rate becomes large; i.e., λ → ∞. It can be readily shown that the characteristic function for shot noise is given by the relationship

$$M_s(j\nu) \equiv E\left(e^{j\nu s}\right)$$
$$= \exp\left(\lambda\int_{-\infty}^{\infty}\left[\exp(j\nu h(\alpha)) - 1\right]d\alpha\right) .$$

Recall that the characteristic function is simply the Fourier transform of the process probability density function. Normalizing the shot noise process to unit variance and zero mean,

$$s^*(t) \equiv [s(t) - E(s(t))] / \sqrt{Var(s(t))}$$
$$= \left[s(t) - \lambda\int_{-\infty}^{\infty} h(\alpha)d\alpha\right] / \sqrt{\lambda\int_{-\infty}^{\infty} h^2(\alpha)d\alpha}$$

the characteristic function of the normalized process becomes

$$M_{s^*}(j\nu) \equiv E\left(e^{j\nu s^*}\right)$$
$$= \exp\left(\lambda\int_{-\infty}^{\infty}\left[\exp\left[\frac{j\nu h(\alpha)}{\sqrt{\lambda\int_{-\infty}^{\infty} h^2(u)du}}\right] - 1\right]d\alpha - j\nu\frac{\lambda\int_{-\infty}^{\infty} h(\alpha)d\alpha}{\sqrt{\lambda\int_{-\infty}^{\infty} h^2(\alpha)d\alpha}}\right) .$$

Using the Maclaurin series expansion for exp(x) = 1 + x + $x^2/2!$ + $x^3/3!$ + …, the above relationship can be simplified as follows:

$$M_{s^*}(j\nu) = \exp\left(\lambda\int_{-\infty}^{\infty}\left[(j\nu)^2\frac{h^2(\alpha)}{2!\lambda\int_{-\infty}^{\infty} h^2(\alpha)d\alpha} + (j\nu)^3\frac{h^3(\alpha)}{3!\left(\lambda\int_{-\infty}^{\infty} h^2(\alpha)d\alpha\right)^{3/2}} + \ldots\right]\right)$$
$$= \exp\left(-\frac{\nu^2}{2} + O(\lambda)\right)$$

where O(λ)→ 0 as λ → ∞. Of course, $\exp(-\nu^2/2)$ is just the characteristic function for an N(0,1) Gaussian random variable x having zero mean and unity variance; i.e.,

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) \Leftrightarrow \exp\left(-\frac{\nu^2}{2}\right).$$

Thus, as expected from the Central Limit Theorem, in the limit of large PRFs, the output of an impulse excited filter tends to a Gaussian random process.

As seen above, the closeness of the approximation to a Gaussian probability density function depends upon the rate λ and the shape of the filter impulse response h(t). As an example, the probability density functions for the envelope of an impulse-excited, ideal bandpass filter are illustrated in Figure 2 as a function of the normalized rate parameter λ/B:

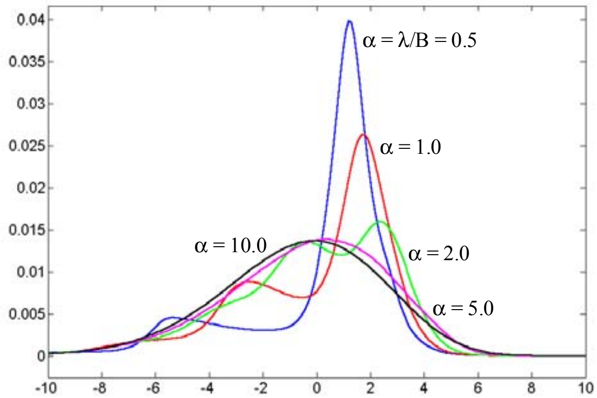

Figure 2. Probability Density Functions for Normalized Envelope of Impulse-excited, Ideal Bandpass Filter

Note that the envelope probability density function is a close approximation to that of a Gaussian variate when the normalized pulse rate λ/B > 5. For the example of a 10 MHz bandwidth Global Positioning Satellite (GPS) receiver, this corresponds to a UWB pulse repetition frequency greater than 50 Mpps.

Thus, in many cases of practical importance, the Gaussian assumption is often invalid.

5. AGGREGATE EFFECTS

Another assertion that has often been made about UWB emissions is that there is no aggregate effect from multiple emitters; i.e., it is only the closest emitter that has an influence on a victim receiver's noise floor [10]. For a single, synchronous, time-division multiplex network architecture, this may indeed be the case as only one emitter is active at any given instant of time, and propagation delays are such that signal overlap is non-existent. However, for simultaneous, uncoordinated networks of emitters, or for other than TDMA architectures (e.g., frequency division multiple access UWB systems), aggregation effects do indeed occur.

It is thus of interest to consider aggregation effects from the perspective of the shot noise process model developed above. In this case, let $\{y_i\}$ be the filter outputs from a collection of N, statistically independent, shot noise random processes $\{x_i\}$ having amplitudes $\{a_i\}$ and rate parameters $\{\lambda_i\}$. That is,

$$y_i(t) = a_i x_i(t) * h_R(t) = \sum_{j=-\infty}^{\infty} a_i u_{-1}\left(t - t_j^{(i)}\right) * \left(p(t) * h_R(t)\right)$$
$$\equiv a_i \sum_{j=-\infty}^{\infty} h\left(t - t_j^{(i)}\right).$$

where $\left\{t_j^{(i)}\right\}$ are the random event times for the Poisson process x_i having rate parameter λ_i. The composite filter output y(t) is given by the sum

$$y(t) = \sum_{i=1}^{N} y_i(t).$$

Now, since the $\{x_i\}$ are statistically independent, so also are the $\{y_i\}$; and the characteristic function for the composite output process y is given by the product of the characteristic functions for the $\{y_i\}$; i.e.,

$$M_y(j\nu) = \prod_{i=1}^{N} M_{y_i}(j\nu) = \prod_{i=1}^{N} \exp\left(\lambda_i \int_{-\infty}^{\infty} \left[\exp(j\nu a_i h(\alpha)) - 1\right] d\alpha\right)$$
$$= \exp\left(\sum_{i=1}^{N} \lambda_i \int_{-\infty}^{\infty} \left[\exp(j\nu a_i h(\alpha)) - 1\right] d\alpha\right).$$

The m^{th} order moments for y(t) can be readily obtained from the characteristic function, noting that

$$\frac{\partial^m}{\partial \nu^m} M_y(j\nu)\Big|_{\nu=0} = (j)^m E\left(y^m\right).$$

Thus, for example, the mean value of the process y(t) is given by the expression

$$E(y(t)) = \sum_{i=1}^{N} a_i \lambda_i \int_{-\infty}^{\infty} h(\alpha) d\alpha.$$

Assuming that $\int_{-\infty}^{\infty} h(\alpha) d\alpha = 0$ (e.g., h(t) is the impulse response of a bandpass filter), one can show that

$$E\left(y^4(t)\right) = 3\left(\sum_{i=1}^{N} \lambda_i a_i^2 \int_{-\infty}^{\infty} h^2(\alpha) d\alpha\right) + \sum_{i=1}^{N} \lambda_i a_i^4 \int_{-\infty}^{\infty} h^4(\alpha) d\alpha.$$

For an ideal bandpass filter, this expression reduces to

$$Energy_{\text{detector output}} \equiv E\left(y^4(t)\right) = 4B^3 \sum_{i=1}^{N} \lambda_i a_i^4 + 12B^2 \left(\sum_{i=1}^{N} \lambda_i a_i^2\right)^2$$
$$= 4B^4 \left[\frac{\sum_{i=1}^{N} \lambda_i a_i^4}{B} + 3\left(\frac{\sum_{i=1}^{N} \lambda_i a_i^2}{B}\right)^2\right].$$

This result is quite similar to that determined earlier for a single shot noise process. Here, however, the rate λ is replaced by a composite "rate" which is a linear combination of the individual $\{\lambda_i\}$. Note that the components of the linear term decay with the fourth powers of the corresponding amplitude parameters; thus, at low pulse repetition frequencies, it is indeed the closest emitter which plays the dominant role.

However, for high PRF emitters, the components of the quadratic term decay with only the squares of the corresponding amplitude parameters. Thus, it is easy to construct examples of physical configurations in which aggregate effects from multiple, high PRF emitters are significantly more deleterious than the effects of the closest, single emitter.

6. CONCLUSIONS

The interaction between a UWB pulse train and a victim receiver was modeled as a shot noise process having rate parameter λ equal to the pulse repetition frequency (PRF) and impulse response that of the convolution of the UWB pulse and the impulse response of the victim receiver front end filter.

For an ideal bandpass filter response, the energy in the square-law detected output process followed a quadratic relationship in λ for $\lambda >> B$, and a linear relationship in λ for $\lambda << B$. This result is consistent with test results performed by Government, academic and commercial laboratories to measure the impact of UWB interference on narrowband systems. In the limit as $\lambda \to \infty$, a simple application of the Central Limit Theorem illustrated the convergence of the probability density function for the output of the victim receiver to that of a Gaussian random variate. However, as shown in the case of an ideal bandpass filter response, this is a reasonable approximation only for $\lambda > 5B$; i.e., only for very high UWB pulse repetition rates. This latter result contradicts the statements often made that UWB “interference” looks like additive, white Gaussian noise to a victim receiver. Finally, aggregate effects from a multiplicity of emitters was again examined from a shot noise perspective; confirming that, in the high PRF case, the effects of multiple emitters are more deleterious than the effects of the closest, single emitter.

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