On "Range-Bandwidth per Joule" for Ultra Wideband and Spread Spectrum Waveforms

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1. Introduction

It has been suggested[1] that an appropriate figure of merit for a low probability of intercept and detection (LPI/D) waveform is the quantity “Range x Bandwidth / Joule”. That is, the further the range, the wider the bandwidth and the less amount of energy used to achieve these values, the more covert is the resultant communications system.

Since the received signal strength from a point source varies as R^{-2} for line-of-sight (LOS) communications and as R^{-4} for mobile communications (due to multipath cancellation), it is perhaps more mathematically appropriate to consider the “figure of merit” as the ratio

$$x \equiv \frac{R^2 B}{E_b} \tag{1}$$

where R is the communications range, B is the signal bandwidth and E_b is the signal energy per bit. (From the range equation, the received signal-to-noise ratio (SNR) is inversely proportional to ξ.) Note that $E_b = P\tau_b$ where τ_b is the bit duration and P is the signal power, assumed to be a constant over the bit duration.

2. Ultra Wideband Waveforms

2.1 UWB LPI/D Figure of Merit

For an ultra wideband (UWB) waveform, the bit duration and the instantaneous bandwidth are related by the Fourier transform relationship between time and frequency. That is, the wide bandwidth in an ultra wideband waveform is produced by pulse duration/shaping and not by spreading with a chipping or hopping sequence as performed by direct sequence and frequency hopping spread spectrum. Thus, the waveform’s time-bandwidth product is given by[2]

$$B\tau_b \approx 1. \tag{2}$$

From the communications range equation, the received signal-to-noise ratio (SNR_R) is given by the expression

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[1] Telephone conversation with Dr. Mark McHenry, DARPA/TTO, 13 March 1998.

[2] For a rectangular pulse, the time-bandwidth product for a 3 dB bandwidth is approximately $B_{3dB}\tau_b \approx 0.886$. The (90% energy) time-bandwidth product is $B_{90\%}\tau_b \approx 1.22$, and the (99% energy) time-bandwidth product is $B_{99\%}\tau_b \approx 2.96$.

$$\mathrm{SNR}_R = \frac{P_T G_T G_R \lambda^2}{(4\pi)^2 R^2 kTB} \tag{3}$$

where P_T is the transmitted power; G_T and G_R are the transmit and receive antenna gains, respectively; λ is the transmission wavelength; k is Boltzmann's constant; and T is the effective system noise temperature.[3]

Combining relationships (2) and (3) into equation (1), one obtains that[4]

$$\xi = \frac{\mu}{(SNR_R)\tau_b} \tag{4}$$

with

$$\mu = \frac{\lambda^2 G_T G_R}{16\pi^2 kT}\,. \tag{5}$$

Note that the quantity μ does not depend upon bandwidth or pulsewidth, but rather is a function of system operational parameters such as antenna gains, center frequency and system noise temperature (e.g., LNA performance). Of course, for any communications system, the received signal-to-noise ratio (equivalently, energy per bit ratio E_b/N_0) determines the resultant bit error rate (BER) for a given modulation strategy.[5]

A cursory examination of equation (4) would suggest that, for a given received signal-to-noise ratio, the LPI/D Figure of Merit can be made arbitrarily large by simply reducing the pulsewidth τ_b. Unfortunately, as τ_b decreases, the peak transmit power also needs to increase in order to keep the energy per received bit a constant.

> ***Thus, a practical limit on the achievable LPI/D Figure of Merit for a UWB communications system is determined by the minimum achievable pulsewidth given a peak power constraint at the transmitter.***

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[3] Note that for any spread bandwidth system, λ is an extended value with a more accurate representation for SNR being an integral over the frequency range of interest; where $P_T = P_T(\lambda)$, $G_T = G_T(\lambda)$, $G_R = G_R(\lambda)$, etc. However, a reasonable estimate of SNR can be found by using the nominal operating wavelength.

[4] In mobile communications, or general multipath propagation, the received signal-to-noise ratio can be shown to be

$$SNR_R = \frac{\lambda^2}{(4\pi R)^2}\left[2\sin\left(\frac{2\pi}{\lambda}\frac{h_T h_R}{R}\right)\right]^2 \frac{G_T G_R P_T}{kTB} \approx \frac{(h_T h_R)^2}{R^4}\frac{G_T G_R P_T}{kTB}$$

where h_T (h_R) are the transmit (receive) antenna heights, respectively. Thus, a more appropriate LPI/D measure may be $\xi = R^4B/E$.

[5] Van Trees, H.L., **Detection, Estimation and Modulation Theory**, Vol. 1, Chapter 4, Wiley, NY, 1968.

Fortunately, for most practical applications, these constraints are very mild as is seen by the following example.

2.2 UWB Examples

2.2.1 Example 1: T1 (1.544 Mb/s) Data Transmission

Consider an ultra wideband communications system requiring a range of 10 miles utilizing one omnidirectional (0 dBi) antenna (e.g., UAV or MAV mounted) and a low gain (+8 dBi) antenna such as an omnidirectional wideband transmission line or patch design. In addition, let the system operate with an L-band center frequency of 1.5 GHz, a 2 dB noise figure and a required (uncoded) Eb/No of +15.6 dB.[6] The required peak power vs. pulsewidth for this hypothetical system is shown in Figure 1 below.

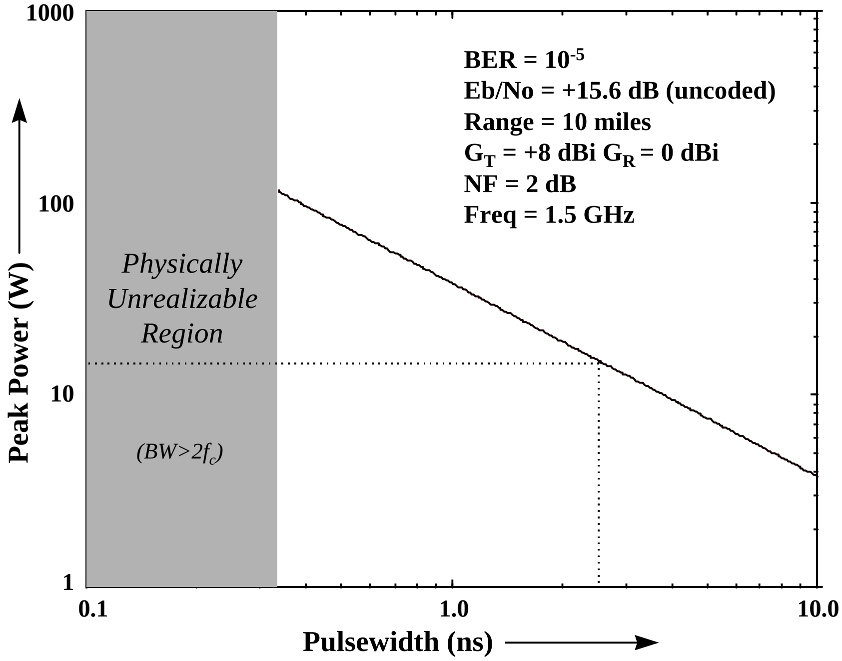

Figure 1. Power vs. Pulsewidth for Example UWB Communications System.

As expected, the requisite peak power increases for a decrease in UWB pulsewidth; however, note that with a 2.5 ns pulse (approximately 400 MHz instantaneous bandwidth), a peak power of approximately 14.5W is required to communicate a distance of 10 miles. [Note that forward error

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[6] For ON-OFF keying (OOK), the bit error probability is given by the relationship

$$P(e) = Q\left(\sqrt{\frac{E_b}{2\mathrm{N}_0}}\right)$$

where Q(x) is the complementary error function defined as $Q(x) = \frac{1}{\sqrt{2p}}\int_x^\infty \exp(-u^2\ /\ 2)du$. Eb/No = +15.6dB corresponds to a 10^{-5} BER.

correction (FEC) coding gain can further reduce this peak power requirement, but only by a small amount.] With currently available components, power levels in excess of several hundred watts peak are now achievable.

The average power of such a UWB emitter can be very low. For example, at a T1 data rate of 1.544 Mb/s, the average output power is approximately 56 mW with a 14.5W peak waveform. This corresponds to a peak power density of 36 nW/Hz, and an average density of only 140 pW/Hz. By way of comparison, a 300 MHz garage door opener has a peak power output of approximately 1 mW and a bandwidth (determined by a dielectric resonator) of roughly 500 kHz. This corresponds to an average (as well as peak) power density of 2 nW/Hz – or 11.5 dB *higher* than for the UWB system which has a communications range of over 10 miles.

2.2.2 Example 2: 32 kb/s Digital (CVSD) Voice Transmission

In a UWB system, the energy per bit does not depend upon the data rate. That is, doubling the rate does not require a corresponding doubling in power to keep the product $P_T\tau_b$ a constant, since τ_b stays fixed. Thus, the peak power is selected to achieve the desired BER; while the average power depends upon the data rate. Lower data rates yield lower average powers because of the reduced duty factor.

Consider reducing the data rate from 1.544 Mb/s to 32 kb/s as used in continuously variable slope detection (CVSD) digital voice. For CVSD voice, a significantly higher BER can be tolerated, e.g. 10^{-3}. At a 10^{-3} BER, the required Eb/No for OOK is now 12.8 dB. Thus, a peak power of only 7.6W is now needed for a 10 mile range. *Note that the peak power was reduced because of an increase in the acceptable BER and* not *because of a decrease in data rate.*

For the 400 MHz bandwidth example above, the average power output at a 32 kb/s data rate would be roughly 600 μW, corresponding to an average power density of only 1.5 pW/Hz.

2.3 Coherent Addition in UWB Processing

The above discussion has been limited to “single pulse” detection of a UWB waveform. However, it is of interest to consider the coherent addition of pulses in an effort to further reduce the LPI/D signature.

From the derivation in Section 2.1, $x = \frac{m}{(SNR_R)t_b}$ where τ_b was defined as the pulsewidth and SNR_R was the single pulse signal-to-noise ratio.

Suppose that a single bit of information is now further subdivided into a sequence (possibly pseudo-randomly generated) of N UWB pulses each of duration τ_b. As before, τ_b sets the ultimate system bandwidth, but the required signal-to-noise ratio can decrease by the factor N since the noise adds noncoherently. Thus, for coherent-UWB

$$x = \frac{Nm}{(SNR_R)t_b} \, . \tag{6}$$

Thus, the effect of utilizing a coherent addition scheme is equivalent to increasing the bandwidth of the UWB waveform by the factor N.

The primary difficulties in achieving coherent-UWB are the following:

a. maintaining coherence over a large number of very short duration pulses can place severe requirements on oscillator stability, particularly at low data rates; and,

b. at high data rates, the improvement factor N becomes limited by physical realizability (For example, at a T1 data rate, N=100 requires a pulse rate of 154 megapulses per second.).

However, the advantages can be substantial. In contrasting UWB with DSSS below, it will become evident that coherent-UWB can significantly outperform any realizable DSSS communications system, even for very low values of N.

As an example, consider the 32 kb/s CVSD voice waveform discussed above. With an N of only 10, the required (uncoded) Eb/No can be reduced to +2.8 dB, requiring a peak power of only 760 mW. Note, however, that the *average* power, and hence the *average* power density, remain the same since the duty cycle is also increased by the same factor N – i.e., 600 μW and 1.5 pW/Hz, respectively.

> NOTE: Various researchers have considered the use of multiple UWB pulses to represent a single bit of information. Unfortunately, in each of these cases, the single pulse detection performance of the receiver was very poor. Pulse addition, therefore, was *necessary* in order to achieve a sufficiently high probability of detection at the receiver. This defeats the purpose of coherent combining for LPI/D and simply results in more energy per unit time being transmitted.
>
> For coherent combining to be of any value for LPI/D, it is therefore essential that a detector be used which has sufficient sensitivity to detect a single UWB pulse with the minimum Eb/No required for reliable communications. In this fashion, each bit of information can be further subdivided into UWB “chips”, each of which by themselves would be deeply buried in the background thermal noise. Coherent combining results in an N-fold SNR enhancement which then enables the combined waveform to trigger a detection event.
>
> MSSI utilizes a sensitive tunnel diode detector which has been shown to permit detection near the thermal noise floor. This detector permits single pulse detection and would form the basis for a coherent combining receiver processor.

3. Spread Spectrum Waveforms

3.1 Direct Sequence Spread Spectrum Figure of Merit

For a direct sequence spread spectrum (DSSS) waveform, a single bit of information is further subdivided into a number of spreading "chips". The ratio of the bit duration τ_b to the chip duration τ_c is denoted as the spreading or processing gain G_P. Thus, the waveform's time-bandwidth product is given by

$$B\tau_c \approx 1. \tag{7}$$

For a DSSS waveform with processing gain G_P, the received signal-to-noise ratio (SNR_R) is given by

$$SNR_R = G_P \frac{P_T G_T G_R \lambda^2}{(4\pi)^2 R^2 kTB}. \tag{8}$$

Thus, combining relationships (7) and (8) into equation (1) noting that $E_b = P_T\tau_b$, one obtains that

$$\xi = \frac{G_P \mu}{(SNR_R)\tau_b} = \frac{\mu}{(SNR_R)\tau_c} \tag{9}$$

where μ is defined in (5).

From relationship (9), for a given received signal-to-noise ratio, the LPI/D Figure of Merit for a DSSS system can be made arbitrarily large by reducing the "chip" width τ_c. For a given bit duration or signaling rate, this corresponds to increasing the system processing gain as expected. Unfortunately, as the processing gain increases, the complexity of the DSSS system also increases.

> ***Thus, a practical limit on the achievable LPI/D Figure of Merit for a direct sequence spread spectrum system is determined by the maximum achievable processing gain given a realizable level of receiver complexity.***

Note that, for a spread spectrum waveform with a given processing gain, the peak power constraint at the transmitter is determined by the bit duration. Note that for any constant envelope waveform (e.g., PSK, GMSK, FSK, etc.), the peak and average power levels are identical.

3.2 DSSS Examples

3.2.1 Example 1: T1 (1.544 Mb/s) Data Transmission

Consider the same example as was considered for the UWB communications system. That is, a 10 mile range is required at 1.5 GHz using an omnidirectional (0 dBi) antenna at one end of the link, and a low gain (+8 dBi) antenna at the other. Assuming a T1 (1.544 Mb/s) binary phase-shift keyed (BPSK) waveform, a 10^{-5} BER corresponds to an Eb/No of +9.6 dB (6 dB better than for an OOK waveform commonly used by UWB systems).

The requisite peak power required for the DSSS can be computed from (8) to be approximately 14.6 mW. The following table provides a summary of the average power densities for the DSSS for different processing gains. Note that the system complexity increases significantly in attempting to achieve the higher processing gains.

Table 1. DSSS Power Densities vs. G_P (1.544 Mb/s System)

Processing Gain G_P (dB)	Bandwidth	Ave. Power Density	Receiver Complexity	Logic Family
10	15 MHz	950 pW/Hz	Low	CMOS
20	150 MHz	95 pW/Hz	Moderate-High	CMOS/ECL
30	1500 MHz	9.5 pW/Hz	Extremely High	GaAs?
33 (theoretical max)	3000 MHz	4.7 pW/Hz	???	???

Note that, on an *average power density basis*, a BPSK DSSS system with a spreading gain of 18.3 dB (100+ MHz BW) is comparable to the 400 MHz bandwidth OOK noncoherent-UWB system. Both systems were designed to achieve the required BER performance.

3.2.2 Example 2: 32 kb/s Digital (CVSD) Voice Transmission

Unlike a UWB system (cf. Section 2.2.2), the energy per bit for a given transmitter peak power *does* depend upon the data rate in a constant envelope system such as DSSS. At a 32 kb/s rate and 10^{-3} BER, the DSSS peak power can be further reduced to 158 μW. Table 2 illustrates the resultant power densities as a function of processing gain for this system design:

Table 2. DSSS Power Densities vs. G_P (32 kb/s System)

Processing Gain GP (dB)	Bandwidth	Ave. Power Density	Receiver Complexity	Logic Family
10	0.32 MHz	500 pW/Hz	Low	CMOS
20	3.2 MHz	50 pW/Hz	Low	CMOS
30	32 MHz	5 pW/Hz	High	CMOS
40	320 MHz	0.5 pW/Hz	Very High	ECL

As seen from the above table, the primary difficulty in achieving high bandwidth (and hence high LPI/D) is the high processing gain required. The higher the processing gain, the more complex the receiver architecture.

4. LPI/D Figure of Merit Comparisons

From the above examples, it is clear that a DSSS system can use substantially less peak power than a noncoherent (single pulse) UWB system to achieve the same communications system performance. From relationships (4) and (8), the relative LPI/D performance is given by

$$\frac{x_{UWB}}{x_{DSSS}} = \frac{(SNR_R^{DSSS})\,t_c^{DSSS}}{(SNR_R^{UWB})\,t_b^{UWB}}\,. \tag{10}$$

For comparable modulation formats[7], the minimum required SNRs are the same and one obtains that

$$\frac{x_{UWB}}{x_{DSSS}} = \frac{t_c^{DSSS}}{t_b^{UWB}}\,, \tag{11}$$

the ratio of chipping time (for DSSS) to pulsewidth (UWB).

For coherent-UWB and DSSS,

$$\frac{x_{UWB}}{x_{DSSS}} = N\,\frac{(SNR_R^{DSSS})\,t_c^{DSSS}}{(SNR_R^{UWB})\,t_b^{UWB}} \tag{12}$$

with N the number of coherently summed UWB pulses.

In the above examples:

T1 (1.544 Mb/s) Data Transmission

The 400 MHz bandwidth OOK UWB waveform has the same LPI/D Figure of Merit as a 100 megachip/second (Mcps) BPSK DSSS waveform. (This takes into account the 6dB loss for OOK vs. BPSK signaling.) The required direct sequence processing gain is thus 18.1 dB. Note that a DSSS processing gain of 24.1 dB would be required to have the same LPI/D performance as an antipodal UWB modulation (+/- pulses).

To compete with an N=10 coherent-UWB signaling scheme, the DSSS system would require a chip rate of 1 Gigachip per second – a very complex design.

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[7] An orthogonal OOK UWB waveform has a theoretical 6 dB performance disadvantage over an antipodal BPSK DSSS signaling scheme. However, antipodal modulation has also been used for UWB. This has the added advantage of removing spectral lines as was shown in Ross, G., R. Price and R. J. Fontana, "The Suppression of Spectral Lines for Improved Covertness in Ultra Wideband (UWB) Transmissions," **Proc. MILCOM 95**, San Diego, CA, November 1995.

32 kb/s Digital (CVSD) Voice Transmission

As in the previous example, a 100 Mcps BPSK DSSS waveform has the same LPI/D performance as a 400 MHz bandwidth UWB signal. In this case, however, the direct sequence processing gain required is 35.0 dB. Note that a DSSS processing gain of 41.0 dB would be required to have the same LPI/D performance as an antipodal UWB modulation (+/- pulses).

Again, to complete with an N=10 coherent-UWB signaling scheme, the DSSS system would require a chip rate of 1 Gigachip per second.

5. Conclusions

An analysis of the LPI/D Figure of Merit, defined as

$$x \equiv \frac{R^2 B}{E_b},$$

demonstrated the equivalence of *noncoherent*-UWB and direct sequence spread spectrum communications for a given bandwidth and Eb/No to achieve a desired bit error rate. (OOK or amplitude shift UWB had a 6 dB disadvantage in Eb/No relative to BPSK DSSS.[8])

The practical limit on the achievable LPI/D Figure of Merit for a UWB communications system was determined by the minimum achievable pulsewidth given a peak power constraint at the transmitter; while the practical limit for DSSS was determined by the maximum achievable processing gain given a realizable level of receiver complexity.

However, it was also shown that *coherent*-UWB has a distinct LPI/D advantage over wideband DSSS systems even for small values of N and appears extremely promising for further development.[9]

Other UWB advantages include:

1. Cost – The primary advantage of UWB over DSSS is significantly lower cost at high levels of LPI/D. For example, a 500 or 1000 MHz instantaneous UWB bandwidth is

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[8] A recent paper – Fontana, R.J., “A Novel Ultra Wideband (UWB) Communications System,” **Proc. MILCOM 97**, Monterey, CA, November 2-5, 1997 – considered the detection-theoretic properties of a high-speed UWB detector which utilizes the charge-sensitive properties of a tunnel diode. The tunnel is a negative resistance device which is extremely responsive to low energy, subnanosecond pulses. In this paper, the author points out the advantages of this type of detector vs. a noncoherent energy detector and demonstrates a signal-to-noise ratio enhancement. This SNR improvement was not considered in the current paper.

[9] The LPI/D advantages of coherent-UWB were also pointed out to the author by Dr. John Betz, The MITRE Corporation. Dr. Betz is a member of the U.S. Government’s Low Probability of Intercept Communications Committee (LPICC).

readily achieved by proper design of transmit/receive filters, amplifier and antenna. The digital electronics is common for all bandwidths and depends solely upon the modulation rate. For DSSS, on the other hand, a 500 or 1000 Mcps system would be expensive to implement, with design costs increasing exponentially with decreasing data rates (due to higher spreading gains).

2. Data rate – At low data rates, it is very difficult to achieve sufficient spreading (processing gain) with a DSSS system to achieve the LPI/D performance of either a noncoherent- or coherent-UWB system. Similarly, at very high data rates (tens of Mb/s and higher), it is difficult to achieve any processing gain with DSSS. Both limitations are due to system realizability and cost constraints. In contrast, UWB system bandwidth does not depend upon the underlying modulation data rate.[10] Thus, there has been considerable interest in UWB for very high data rate applications such as real-time video transmission and multi-terminal networking.

3. Multipath immunity – Modern high-speed UWB detectors are able to trigger close to the leading edge of the received pulse. As a consequence, multipath returns which occur later than the pulse duration do not affect the received signal strength. For example, with a 2.5 ns UWB pulse, any return due to path differentials larger than 2.5 feet ($c \approx 1$ ft/ns) is effectively gated out. Thus, UWB has been found to be extremely effective in in-building, vehicle-to-vehicle and vehicle-to-roadside communications.

   DSSS can also have good multipath immunity provided the spread bandwidth exceeds the reciprocal multipath delay. In this case, immunity is provided by the orthogonality of the PN code with its time-shifted replica. Unfortunately, most commercial applications of DSSS have been restricted to the three ISM bands (902-928, 2400-2483.5 and 5725-5850 MHz) where the available bandwidths are limited.

4. Dual Use – A UWB communications waveform is essentially indistinguishable from a low power UWB radar pulse. As a consequence, much of the same electronics can be used for both communications and high resolution radar.[11] In addition, since UWB detectors are capable of response times faster than 100 ps, the pulses can also be used for high precision geolocation.

   DSSS has also been used in precision ranging applications; however, precision is a function of spreading bandwidth which becomes very expensive for resolutions finer than a few feet.

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[10] This independence of UWB spectrum occupancy on data rate is significant for another reason as well. DSSS has a familiar sin(x)/x structure which is readily exploited for identification (e.g., chip rate, carrier frequency, etc.); whereas UWB has a relatively "noise-like" structure which is more difficult to exploit.

[11] For example, in a Phase II SBIR program for the Navy's Program Executive Office for Unmanned Aerial Vehicles and Cruise Missiles, MSSI developed a common UWB radar/communications module for radar altimeter, collision/obstacle avoidance and data link functions.